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Teaching and Learning

P3 practice paper (Activity 5)

1. The point $P(7, -2)$ lies on the curve with equation $y = f(x)$.

State the coordinates of the image of P under the transformation represented by the curve with equation

(a) $y = f(x + 4) + 1$

(2)

(b) $y = f(-2x) + 7$

(2)

2. (a) Using the identity for $\cos(A \pm B)$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A$$

(2)

(b) Hence find, using calculus, the exact value of

$$\int_0^{\frac{\pi}{4}} 3\sin^2 2x \, dx$$

(3)

3. Guinea pigs and rabbits were introduced onto an island at the same time.

The number of guinea pigs, G, t months after they were introduced onto the island is modelled by the equation

$$G = a + 60e^{-0.05t}$$

where a is a positive constant.

The number of rabbits, R, t months after they were introduced onto the island is modelled by the equation

$$R = 100 + 80e^{0.05t}$$

Given that there were twice as many guinea pigs as rabbits introduced onto the island,

(a) find the value of a.

(2)

When $t = T$, the number of rabbits on the island is equal to the number of guinea pigs on the island.

Using these models,

(b) find the value of T, giving your answer to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

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In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

4. Solve, for $0 \leqslant \theta < 2\pi$, the equation

$$3\sin 2\theta + 5\cos 2\theta = 4$$

giving each answer to 2 significant figures.

(7)

5. A function f is defined by

$$\mathrm{f}(x) = \frac{7}{x+3} - \frac{5x+22}{x^2+7x+12} \qquad x \in \mathbb{R},\ x > -3$$

(a) Show that f(x) can be written in the form $\frac{A}{Bx+C}$ where A, B and C are constants to be found.

(3)

(b) Hence find f^{-1}

(3)

(c) Solve $\mathrm{ff}(x) = \frac{2}{5}$

(3)

6.

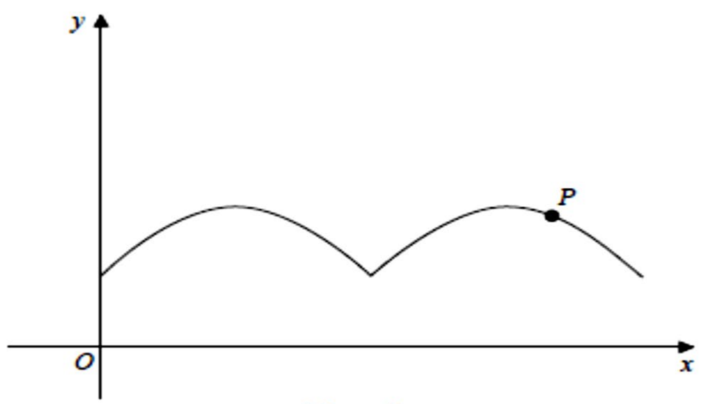

Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = |\sin x| + 1 \qquad 0 \leqslant x \leqslant 2\pi$$

The point $P(a, b)$ lies on the curve and is shown on Figure 1.

Given that the gradient of the curve at P is $-\frac{1}{2}$

(a) find the exact value of a and the exact value of b.

(4)

A straight line with positive gradient passes through P.

Given that the straight line intersects the curve at exactly three distinct points,

(b) find the range in values of the gradient of the line.

(3)

7. A curve has equation

$$y = e^{2\sqrt{3}x}\cos 2x \qquad 0 < x < \pi$$

(a) Find $\frac{dy}{dx}$ **(2)**

(b) Hence, using algebra and showing your working, find the exact coordinates of the stationary points of the curve.

(Solutions based entirely on graphical or numerical methods are not acceptable.) **(6)**

8. Liam monitored the population of a small country over a 10-year period.

The population, P, measured in thousands of people, is modelled by the equation

$$P = ab^{-t}$$

where a and b are constants and t is the number of years since monitoring began.

(a) Show that this equation can be expressed in the form

$$\log_{10} P = \log_{10} a - t\log_{10} b$$

(1)

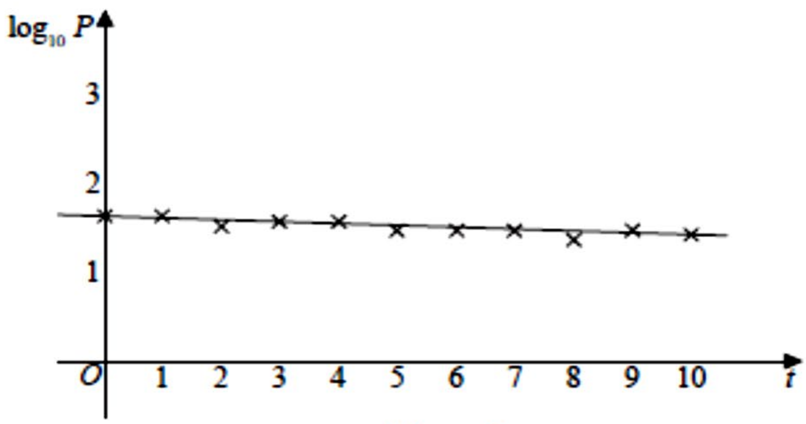

Figure 2

Figure 2 shows a line of best fit for values of t and $\log_{10} P$

The line of best fit passes through points (0, 1.6) and (10, 1.4)

Using this information,

(b) find the value of a and the value of b, giving each answer to 4 significant figures. **(4)**

Hence, according to the model,

(c) find the rate at which the population was changing exactly 8 years after monitoring began. **(3)**

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9. A curve has equation

$$x = \frac{\sin y - \cos y}{\cos y + \sin y} \qquad -\frac{\pi}{4} < y < \frac{3\pi}{4}$$

(a) Show that the equation of the curve can be written as

$$x = \frac{-\cos 2y}{1 + \sin 2y} \qquad -\frac{\pi}{4} < y < \frac{3\pi}{4}$$

(3)

(b) Hence, or otherwise, show that

$$\frac{dx}{dy} = \frac{2}{1 + \sin 2y}$$

(2)

A point $P(x, y)$ lies on the curve.

Given that at P

- $\frac{dy}{dx} = \frac{1}{4}$
- $y < 0$

(c) find the exact coordinates of P

(5)

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10.

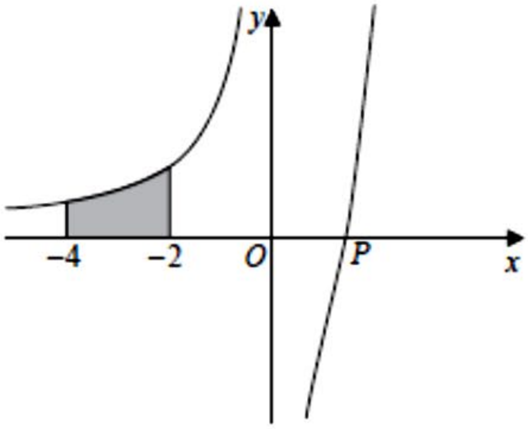

Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = e^{2x-3} - \frac{4}{3x} \qquad x \in \mathbb{R}, x \neq 0$$

The curve crosses the x-axis at the point $P(\alpha, 0)$.

(a) Show that

$$\alpha = \frac{1}{2}\left(\ln\left(\frac{4}{3\alpha}\right) + 3\right)$$

(2)

The iteration formula

$$x_{n+1} = \frac{1}{2}\left(\ln\left(\frac{4}{3x_n}\right) + 3\right)$$

is used to find an approximation to α.

(b) Taking $x_0 = 2$ find the value of x_1 and the value of x_5

Give each answer to 4 decimal places.

(2)

(c) Using a suitable interval and a suitable function that should be stated, show that to 3 decimal places α is 1.456

(2)

The finite region, shown shaded in Figure 3, is bounded by the curve, the line with equation $x = -4$, the x-axis and the line with equation $x = -2$

(d) Using integration find, in simplest form, the exact area of the shaded region.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)